30th Annual Conference of the Ramanujan Mathematical Society
Symposium on Number Theory
Abstracts of Talks

May 15, 2015

11:30 AM - 12:10 PM

On the zeros of General L-functions
K. Srinivas, Institute of Mathematical Sciences, Chennai

Abstract: Selberg defined a class of functions (popularly known as Selberg class) having properties similar to the Riemann zeta-function. Selberg class is full of open problems. In this talk, we shall discuss some results related to the zeros of functions in this class.

12:15 PM - 12:55 PM

Large subsets of the integers
D. Surya Ramana, Harish-Chandra Research Institute, Allahabad

Abstract : The talk is centered around the following somewhat vaguely phrased question : do large but otherwise arbitrary subsets of interesting subsets of the integers such as the primes, the squares etc retain some of their properties ? For instance, every large enough natural number is a sum of no more than four prime numbers. Then one may ask, given a subset of positive density in the primes, what is the smallest number of summands required to represent all large enough natural numbers as a sum of primes from this dense subset.

We will begin our talk by giving a number of other examples, some conjectures and some theorems, that clarify the meaning of this question and then proceed to describe our recent work that falls within this theme.

2:00 PM - 2:40 PM

Non-homogeneous Problems: Conjectures of Minkowski and Woods
Madhu Raka, Panjab University, Chandigarh

Abstract: A classical conjecture in Geometry of Numbers attributed to H. Minkowski states that if

$$L_i = a_{i1}x_1 + \cdots + a_{in}x_n,\ 1 \leq i \leq n$$

are n real linear forms in n variables $x_1, \ldots, x_n$ and of determinant $\Delta = \det(a_{ij}) \neq 0$, then given any real numbers $c_1, \ldots, c_n$, there exists integers $x_1, \ldots, x_n$ satisfying

$$| (L_1 + c_1) \cdots (L_n + c_n) | \leq \frac{1}{2^n} | \Delta | .$$

Equality is necessary if and only if after a suitable unimodular transformation the linear forms L_i have the form $2c_ix_i$ for $1 \leq i \leq n$.

In this talk, we shall give the history of the Conjecture and the recent progress to settle it for $n \leq 9$. Following Remak-Davenport Approach and using a result of McMullen [2005], it is sufficient to prove Woods' Conjecture for lattices reduced in the sense of Korkine and Zolotareff. We shall illustrate the difficulties we face in proving Woods' Conjecture for higher dimensions. For $n \geq 10$, we obtain estimates to Minkowski's Conjecture improving the previous known results.

2:45 PM - 3:25 PM

Dedekind's Theorem on Splitting of Primes: 137 Years of Journey
Sudesh K. Khanduja, Indian Institute of Science Education and Research, Mohali

Abstract: Let $K = \mathbb{Q}(\theta)$ be an algebraic number field with $f(x)$ as the minimal polynomial of the algebraic integer θ over $\mathbb{Q}$. Let p be a rational prime.Let

$$\bar{f}(x) = \bar{g}_1(x)^{e_1} \dots \bar{g}_r(x)^{e_r}$$

be the factorization of $\bar{f}(x)$ as a product of powers of distinct irreducible polynomials over $\mathbb{Z}/p\mathbb{Z}$, with $g_i(x)$ monic polynomials belonging to $\mathbb{Z}[x]$. In 1878, Dedekind proved if p does not divide the index of the subgroup $\mathbb{Z}[\theta]$ in A_K, then $pA_K = \wp_1^{e_1} \dots \wp_r^{e_r}$, where $\wp_1, \dots, \wp_r$ are distinct prime ideals of A_K, $\wp_i = pA_K + g_i(\theta)A_K$ with residual degree of $\wp_i/p$ equal to $\deg g_i(x)$ for all i. In 2008, we proved that converse of Dedekind's theorem holds, i.e. if for a rational prime p the decomposition of pA_K satisfies the above three properties, then p does not divide $[A_K : \mathbb{Z}[\theta]]$. Dedekind also gave a simple criterion known as Dedekind Criterion to verify when p does not divide $[A_K : \mathbb{Z}[\theta]]$. We will also discuss the Dedekind Criterion and its generalization. In 2014, we have proved the analogue of Dedekind's theorem for finite extensions of valued fields of arbitrary rank as well as its converse.

May 16, 2015

11:30 AM - 12:20 PM

Twin Primes
M. Ram Murty, Queen's University, Ontario

Abstract: The twin prime problem is the assertion that there are infinitely many distinct primes p, q with $|p - q| = 2$. This is still an open problem. In May 2013, Yitang Zhang surprised the world by proving that there are infinitely many pairs of distinct primes p, q such that $|p-q| < 70$ million. Until Zhang's work, no bound was known. After Zhang's paper, this bound has been improved and in November 2013, Maynard and Tao gave a simplified proof of Zhang's theorem with better numerical results. The bound now is 246. We will survey these developments and outline a new higher rank sieve. As a consequence, the Zhang-Maynard-Tao theorem is deduced as a special case of a more general result. This general result leads

to improvements on earlier work of Heath-Brown on prime k-tuples. This is joint work with Akshaa Vatwani.

12:25 PM - 1:05 PM

Gaps between Hecke eigenvalues
Kaneenika Sinha, Indian Institute of Science Education and Research, Pune

Abstract: We will derive some new results about the behaviour of gaps between the eigenvalues of the Hecke operator acting on modular cusp forms. This is a joint work with Sudhir Pujahari.

2:00 PM - 2:40 PM

Four Exponential Conjecture
R. Thangadurai, Harish-Chandra Research Institute, Allahabad

Abstract. Let x and y be Q-linearly independent complex numbers and a and b be Q-linearly independent complex numbers. Then Four Exponential Conjecture says that among the four numbers $\exp(ax), \exp(bx), \exp(ay), \exp(by)$, at least one of them is transcendental. We shall report the recent work on this conjecture.

2:45 PM - 3:25 PM

Uniform cyclotomy, density of primes and zeta functions
S. A. Katre, SP Pune University, Pune

Abstract: In this talk we shall consider the condition of uniform cyclotomy, and get the density of primes which satisfy this condition. We then find the zeta function of the projective curve $aY^e = bX^e + cZ^e$ and hence the divisor class number of this curve in the case of uniform cyclotomy.