30th Annual Conference of the Ramanujan Mathematical Society

Symposium on Topology

Abstracts of Talks

May 15, 2015

11:30 AM - 12:10 PM

Walkup class $\mathcal{H}^4$ and tight triangulations of 3-manifolds

Basudeb Datta, Indian Institute of Science, Bangalore

Abstract: A triangulated $(d+1)$-manifold with non-empty boundary is said to be *stacked* if all its interior faces have dimension $\geq d$. A closed triangulated d-manifold M is said to be *stacked* if $M = \partial N$ for some stacked triangulated $(d+1)$-manifold N. A triangulated manifold is said to be *locally stacked* if each vertex link is a stacked sphere.

For $d \geq 3$, we recursively define the class $\mathcal{H}^d(k)$ as follows. (a) $\mathcal{H}^d(0)$ is the set of stacked $(d-1)$-spheres. (b) A triangulated d-manifold Y is in $\mathcal{H}^d(k+1)$ if it is obtained from a member of $\mathcal{H}^d(k)$ by a combinatorial handle addition. (c) The *Walkup's class* $\mathcal{H}^d$ is the union $\mathcal{H}^d = \bigcup_{k\geq 0} \mathcal{H}^d(k)$.

For a field $\mathbb{F}$, a simplicial complex X is called $\mathbb{F}$-*tight* if (i) X is connected, and (ii) for all induced subcomplexes Y of X and for all $0 \leq j \leq \dim(X)$, the morphism $H_j(Y;\mathbb{F}) \to H_j(X;\mathbb{F})$ induced by the inclusion map $Y \hookrightarrow X$ is injective. For $d \geq 3$, a triangulated d-manifold M is called *tight neighborly* if $\beta_1(M;\mathbb{Z}_2) = \binom{f_0(M)-d-1}{2} / \binom{d+2}{2}$.

In this talk we would like to discuss the following recent results which are joint work with B. Bagchi and J. Spreer; B. A. Burton, N. Singh and J. Spreer; S. Murai.

Theorem 1. *Let M be a closed triangulated 3-manifold. If M is tight neighborly then M is locally stacked and $\mathbb{F}$-tight.*

Theorem 2. *For $d \geq 2$, a connected triangulated d-manifold M without boundary is stacked if and only if $M \in \mathcal{H}^{d+1}$.*

Theorem 3. *If a triangulated closed 3-manifold M is tight with respect to some field $\mathbb{F}$ with* $\mathrm{char}(\mathbb{F}) \neq 2$ *then M is stacked.*

As a consequence of the above results we obtain:

Corollary. *Let M be a connected, orientable, closed triangulated 3-manifold and $\mathbb{F}$ be a field with* $\mathrm{char}(\mathbb{F}) \neq 2$. *Then the following are equivalent.* (i) *M is tight-neighborly.* (ii) *M is a neighborly member of $\mathcal{H}^4$.* (iii) *M is neighborly and stacked.* (iv) *M is $\mathbb{F}$-tight.*

12:15 PM - 12:55 PM

Deformation cohomology of algebras over operads
Anita Naolekar, Indian Statistical Institute, Bangalore

Abstract: An algebra of a certain type is usually defined by gen- erating operations and relations. Given a type of algebras there is a notion of free algebra over a generic vector space V, say $\mathcal{P}(V)$. Viewed as a functor from the category $Vect$ of vector spaces to itself, $\mathcal{P}$ is equipped with a monoid structure, that is a transformation of functors $\gamma : \mathcal{P} \circ \mathcal{P} \to \mathcal{P}$, which is associative, and another one $\eta : I \to \mathcal{P}$ which is a unit. The existence of this structure follows readily from the universal properties of free algebras. Such a data $(\mathcal{P}; \gamma; \eta)$ is called an algebraic operad. On the other hand, any operad gives rise to a type of algebras, the $\mathcal{P}$-algebras. Given a $\mathcal{P}$-algebra structure, we introduce the cohomology of this structure, and finally, after defining deformations of this structure, we show that this cohomology controls the deformations.

2:00 PM - 2:40 PM

On trivialities of characteristic classes over suspension space
Ajay Thakur, Indian Statistical Institute, Bangalore

Abstract: A CW-complex X is said to be W-trivial if for any vector bundle ξ over X, the total Stiefel-Whitney class $W(\xi) = 1$. It is a theorem of Atiyah-Hirzebruch that the k-fold suspension $\Sigma^k X$ of any CW-complex X is W-trivial if $k > 8$. A related notion is that of C-triviality. A CW-complex X is said to be C-trivial if for any complex vector bundle η over X, the total Chern class $C(\eta) = 1$. In this talk we shall state some general results and investigate when the iterated suspensions of projective spaces are W-trivial and C-trivial.

2:45 PM - 3:25 PM

Equivariant cobordism classes of Milnor manifolds
Swagata Sarkar, Indian Institute of Science, Bangalore

Abstract: Let $\mathcal{N}_*$ be the unoriented cobordism algebra, let $G = (\mathbb{Z}_2)^n$, and let $Z_*(G)$ denote the equivariant cobordism algebra of G-manifolds with finite stationary point sets. Further, let $\epsilon_* : Z_*(G) \to \mathcal{N}_*$ be the homomorphism which forgets the G-action. A cobordism class $[M] \in Z_*(G)$ is said to be indecomposable if it cannot be expressed as the sum of products of lower dimensional cobordism classes. Indecomposable classes generate the cobordism algebra $Z_*(G)$. We discuss a sufficient criteria for ‘indecomposability’. Using the above mentioned criterion, we show that the classes of Milnor manifolds (degree 1 hypersurfaces in $\mathbb{R}P^m \times \mathbb{R}P^n$) give non-trivial, indecomposable elements in $Z_*(G)$ in degrees up to $2^n - 5$. Moreover, we show that in most cases these elements can be arranged to be in $Ker(\epsilon_*)$. We also give a lower bound for the number of linearly independent elements in $Z_d((\mathbb{Z}_2)^k)$, where $1 \leq d \leq 2^{k-i+1} - 5$. (This talk is based on joint work with Samik Basu and Goutam Mukherjee.)

May 16, 2015

11:30 AM - 12:10 PM

Smooth Structures on a fake real projective space

Ramesh K., Indian Statistical Institute, Kolkata

Abstract: In this talk, we first study the group of smooth homotopy m-spheres Θ_m due to Michel Kervaire and John Milnor (1963). By using the group Θ_7, we classify, up to diffeomorphism, all closed manifolds homeomorphic to the real projective 7-space $\mathbb{R}P^7$. We also show that if M is a closed smooth manifold homotopy equivalent to $\mathbb{R}P^7$, then M has exactly 56 distinct differentiable structures up to diffeomorphism.

12:15 PM - 12:55 PM

Multicurves and primitivity in $Mod(S_g)$

Kashyap Rajeevsarathy, Indian Institute of Science Education and Research, Bhopal

Abstract: Let S_g be a closed orientable surface of genus $g = 2$, and let $\mathcal{C} = \{c_1, \ldots, c_m\}$ be multicurve in S_g. Since the Dehn about any two distinct curves in $\mathcal{C}$ commute in the mapping class group $Mod(S_g)$, the Dehn twist $t_{\mathcal{C}} = t_{c_1} t_{c_2} \ldots t_{c_m}$ about $\mathcal{C}$ is well-defined.

A root h of $t_{\mathcal{C}}$ may either fix the curves in $\mathcal{C}$ or permute them, and accordingly h will be classified as permuting or nonpermuting. We will describe the geometric construction of permuting and nonpermuting roots, and derive equivalent conditions for their existence. We will also discuss some recent results pertaining to this problem. Finally, we will give a deeper insight into the problem of general primitivity in $Mod(S_g)$.

2:00 PM - 2:40 PM

Contact Structures and Heegaard Floer Homology

Dheeraj Kulkarni, Ramkrishna Mission Vivekananda University, Belur

Abstract: There is a dichotomy between contact structures – tight and overtwisted. Classification of overtwisted contact structures up to isotopy, due to Eliashberg, is well-known. There have been few results towards classification of tight contact structures. However, in general, given a contact structure it is difficult to know whether it is tight or not. In this talk, we focus on the role of certain element, called as contact invariant, of Heegaard Floer homology group associated with a given contact structure in understanding tightness. We discuss success and limitations of contact invariant in detecting tightness of contact structures. In this direction, I will also present a recent result from joint work with James Conway and Amey Kaloti. We will briefly survey relevant notions from Contact Topology and Heegaard Floer Homology in first half of the talk. The audience will have a glimpse of “soft” meadows and “rigid” rocky features in the terrain of Contact Topology.